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February 4, 2019

Mathematics of football free kicks

Analytic Mechanics
FYGB08

Abstract

A direct free kick is a method of restarting play in a game of football that is awarded to a team following a foul from the opposing team. Free kicks are situations that professional footballers have been practicing daily, with world class coaches, for the majority of their life, however the conversion rate of a free kick is somewhere between $0 - 10\%$ depending on where the free kick is taken. In this report the physics of free kicks was investigated by analyzing a ball of which gravitational force, drag force and the Magnus force were acting on the ball. The purposes of the report were to both describe the trajectory of a football and to investigate the low conversion rate of direct free kicks. The mathematics of free kicks was implemented in MatLab and simulations are consistent with the physical intuition experienced when watching a game. It was concluded that small deviations of the initial conditions made a great impact on the trajectory.

Contents

1 Introduction

Sport have caught the interest of a large number of people all over the planet. One part that make sport interesting is that it's often unpredictable and the margins of error are small. One of these situation is the free kick in football. Free kicks are situations that professional footballers have been practicing on a daily basis, with world class coaches, for the majority of their life. It's therefore interesting that the conversion rate (number of goals / free kick) is very low. In all top European leagues from 2009 and 2017 the conversion rates of direct free kicks are somewhere between 0% and 10% depending on where the free kick is located in relation to the goal. [7] In this report the underlying physics of the low conversion rate will be investigated by analyzing the physics of direct free kicks.

A direct free kick is a method of restarting play in a game of football that is awarded to a team following most types of fouls. In a direct free kick, the fouled team is entitled to freely kick the ball from the spot of the foul, with opponents required to be at least 7 m from the ball. The kicking team may score a goal directly from a direct free kick, ie scoring without having the ball touch another player. An indirect free kick on the other hand is a free kick that touches another player in order. If a player commits a foul within his/her own penalty area, a penalty kick is awarded instead. The penalty area and other regions on a football pitch are illustrated in figure 1 with their standard measurements. The defending team do usually place some of its players next to each other between the

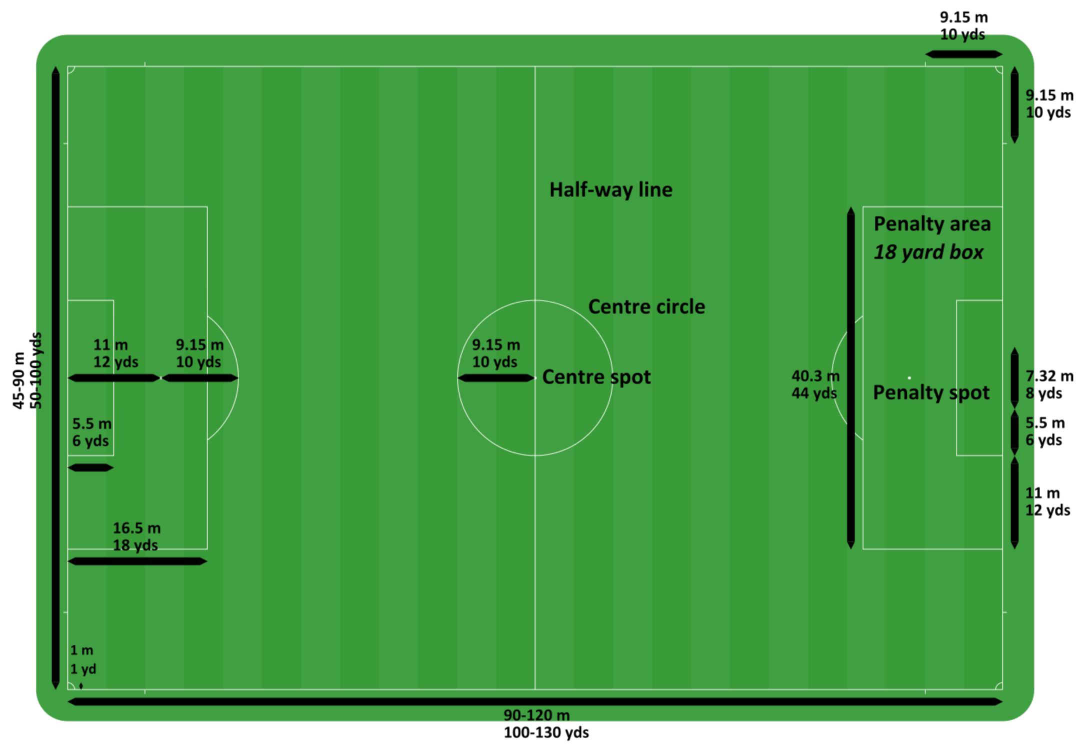

Figure 1: Standard pitch measurements.
Figure adapted from [6]

ball and the goal. These players are forming what in football is described as a wall. The wall is a term that will be used throughout this report. The "Fédération Internationale de Football Association" (FIFA) is the international federation of football and all regulations, lengths and masses used in this report are consistent with FIFA regulations.

2 Theory

2.1 Declaration of variables and notation

The variables and notation declared in table 1 will be used throughout the report. The ball can in general experience six degrees of freedom, however the rotation of the ball was restricted to be inwards to the goal. This will leave the system with four degrees of freedom, movement in x,y and z direction and rotation about the z-axis. There are mainly four forces acting on a football, gravitational force $\boldsymbol{F}_g$, drag force $\boldsymbol{F}_D$, lift force $\boldsymbol{F}_L$ and a sideways force $\boldsymbol{F}_S$.

Variable	Description	Variable	Description
Vectors		**Scalars**	
$\boldsymbol{F}_g$	Gravitational force	P	Pressure
$\boldsymbol{F}_D$	Drag force	v	Velocity of the ball
$\boldsymbol{F}_L$	Lift force	ρ	Density of air
$\boldsymbol{F}_S$	Sideways force	A	Area of the ball
$\boldsymbol{a}$	Acceleration	r	Radius of the ball
$\boldsymbol{g}$	Gravitational constant on earth	C_D	Drag coefficient
$\boldsymbol{e}_{q_i}$	Unit vector in the direction	C_L	Lift coefficient
	of the generalized coordinate q_i	C_L	Lift coefficient
		C_S	Sideways coefficient
		Re	Reynolds number

Table 1: Table of variables used throughout the report.

These forces are illustrated in figure 2. The sideways force can't be visualized in a two dimensional figure since it's acting perpendicular to the velocity vector and the lift force. These forces will be described in more detail in the following sections.

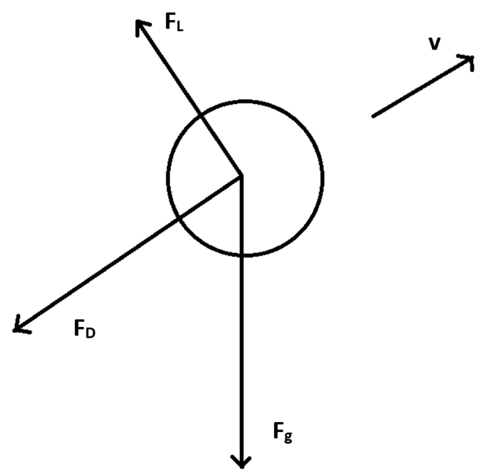

Figure 2: The direction of the most significant forces acting on a ball in movement.

2.2 Gravitational force

Sir Isaac Newton revolutionized physics when he stated his laws of motion. In Newton's second law of motion he formulated that the acceleration $\boldsymbol{a}$ of an object (produced by a net force $\boldsymbol{F}$) is directly proportional to the magnitude of the net force, in the same direction as the net force. The acceleration is also inversely proportional to the mass m of the object. By rearranging the terms, Newton's second law of motion can be described as

$$\sum_i \boldsymbol{F}_i = \sum_i m_i \boldsymbol{a}_i. \tag{1}$$

Although Albert Einstein described gravitation as a consequence of the curvature of spacetime, in most other experiments on earth, it's sufficient to use Newtonian physics and view gravity as a force and time as a parameter. The gravitational pull on objects on earth is described by $\boldsymbol{g} = -9.81\boldsymbol{e}_z \, m/s^2$. The gravitational force on a soccer ball can therefore be expressed as

$$\boldsymbol{F}_G = m\boldsymbol{g} \tag{2}$$

where m is the mass of the ball. In Newton's first law he define that a body act upon by zero forces shall remain at rest, described mathematically by $\sum_i \boldsymbol{F}_i = 0$. This law explain that a body at constant velocity will stay at constant velocity unless a force acts on it. These laws are fundamental for the kinematics of flying objects.

2.3 Drag force

Drag is a non-conservative force and will continuously reduce the energy of the ball until its velocity reaches zero. The drag force on a soccer ball can be expressed as

$$\boldsymbol{F}_D = \frac{\rho A v^2 C_D}{2} \cdot \boldsymbol{e}_{-v}. \tag{3}$$

The drag coefficient is a function of the Reynolds number and the roughness of the surface. The Reynolds number is a number that describe the relative magnitude of pressure drag and viscous drag, caused by the viscosity ν, as

$$Re = \frac{\text{Pressure Drag}}{\text{Viscous Drag}} = \frac{2|\boldsymbol{v}|r}{\nu}. \tag{4}$$

The Reynolds number is important to investigate when performing calculations on fluids or aerodynamics since, at a critical number, the Reynolds number will change dramatically as the flow changes between laminar and turbulent flow. A rough ball cause turbulence at lower Reynolds number than a smooth ball. For a soccer ball the transition from laminar flow to turbulent flow occurs at approximately 10-14 m/s depending on the design of the football. [1, 2, 3] All free kicks investigated in this report have high velocity and therefore there will be no dramatic change from laminar to turbulent flow to account for. If $C_D > 0$ the force will slow the ball down whereas $C_D = 0$ imply that the force is not present in the simulation. C_D is bound to be positive since a negative force would increase the velocity of the ball which is a non-physical phenomenon.

2.4 The Magnus effect

Bernoulli's theorem state that the total mechanical energy of the flowing fluid, with all of its components (potential, kinetic, flow etc.) must be conserved according the law of conservation of energy for ideal fluids. The Bernoulli equation for a fluid particle in a streamline can be derived by applying Newtons second law, mentioned earlier in equation 2. When net frictional forces and heat transfer are negligible, Newtons' second law along the streamline can be expressed according to

$$-\,\mathrm{d}P - \rho g \mathrm{d}z = \rho v \mathrm{d}v. \tag{5}$$

Rewriting this expression give

$$\frac{\mathrm{d}P}{\rho} + \frac{1}{2}\mathrm{d}(v^2) + g\mathrm{d}z = 0 \tag{6}$$

or in integral form

$$\int_{P_1}^{P_2} \frac{\mathrm{d}P}{\rho} + \int_{v_1}^{v_2} \frac{1}{2}\mathrm{d}(v^2) + \int_{z_1}^{z_2} g\,dz = 0. \tag{7}$$

Evaluating these integrals along the streamline from point 1 to point 2 give

$$\frac{P_1}{\rho} + \frac{v_1^2}{2} + g z_1 = \frac{P_2}{\rho} + \frac{v_2^2}{2} + g z_2. \tag{8}$$

Equation 8 is referred to as the Bernulli equation between two points 1 and 2 on the same streamline when steady, incompressible flow is investigated.[4] The Magnus effect can be viewed as a special case of the Bernulli's theorem. The Magnus effect is named after H.G. Magnus who investigated this effect experimentally in the mid 1850s. He discovered that it's this effect that cause the balls to depart from its straight trajectory when subjected to spin due to the pressure difference induced by the spin of the ball. The velocity of the fluid will change since some parts of the fluid will follow the spin of the ball. The side of the ball where the speed of the fluid increases will also experience pressure reduction in accordance with Bernulli's equation (equation 8). Higher pressure of the low speed fluid forces the ball in the direction of the low pressure / high velocity region on the opposite side. [10]

The spin of the ball will give rise to both a lift force and a sideways force depending on the rotation of the spin. The lift force $\boldsymbol{F}_L$ is perpendicular to the drag force and the sideways force $\boldsymbol{F}_S$ can be described mathematically as

$$\begin{aligned} \boldsymbol{F}_L &= \frac{\rho A v^2 C_L}{2}\boldsymbol{e}_l \\ \boldsymbol{F}_S &= \frac{\rho A v^2 C_S}{2}\boldsymbol{e}_s \end{aligned} \tag{9}$$

where $\boldsymbol{e}_l$ is the unit vector perpendicular to $\boldsymbol{e}_v = \boldsymbol{v}/v$ which is the unit vector in the direction of the ball. The sideways force acting according to $\boldsymbol{e}_s = \boldsymbol{e}_l \times \boldsymbol{e}_v$. C_L and C_S are the dimensionless lift and sideways coefficients respectively. [3, 8] If $C_S > 0$ the object will depart to the left and $C_L < 0$ the object depart to the right. If $C_L > 0$ the object experience lift whereas $C_L < 0$ will make the ball fall quicker. For $C_L = C_S = 0$ these forces vanish and are not present in the simulations.

3 Method

Initially different models to describe the trajectory of a ball with specific initial conditions where investigated. After this investigation the best approximation was used in order to validate the conversion rate of free kicks. All free kick simulations where performed at the same distance to goal with only left spin of the ball for computational reasons that will be discussed later. In order to validate the conversion rate the simulations need to be as realistic as possible and therefore characteristic properties of the game was modeled, such as a wall, a goal and a goalkeeper. These models have been designed through trial and error and this process will be described in a later section. When all properties of the game worked sufficiently great Monte Carlo simulations for six different ball trajectories were performed with an error in the initial velocities that was varied in order to estimate the uncertainty in the forward. The balls that scored hit the goal at different locations and this was investigated as a final part of this experiment. The goal was divided into 6x3 equally sized regions. The regions in the two top corners are named *top corners*, the regions that are in the directs neighbourhood of the top corners are named *neighbours* and all other regions are named *distant*. In order to clarify these are illustrated in figure 3. It was decided to only investigate the trajectory of a typical right footed forward since this is more common, hence only $C_S \geq 0$ have been investigated.

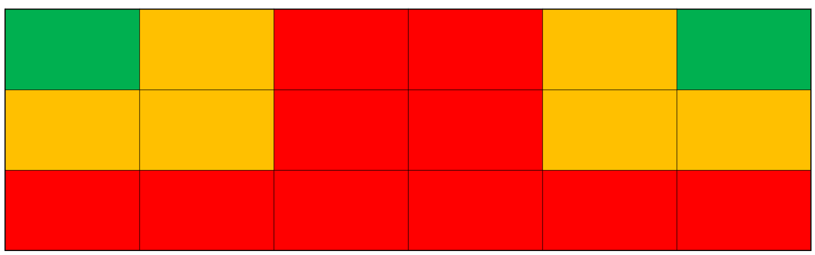

Figure 3: Different regions of a goal. Green: Top corners, Yellow: Neighbours, Red: Distant.

Constants

The constants used in this report are declared in this section. The mass and the radius of the football, $m = 0.444$ kg and $r = 0.11$ m. The area of the ball is given by $A = \pi r^2$. All specifications on the ball and all distances on the imaginary football pitch are official FIFA standards. [5] The ball is approximated to be spherically symmetric. The density and viscosity of air at room temperature are approximated to be constant, $\rho = 1.225$ kg/m^3 and $\nu = 1.5 \cdot 10^{-5}$ m^2/s. [12] In order to make the MatLab calculations more tidy, $\beta = \frac{\rho A}{2m}$ was defined. The goal fulfills FIFA standards of 7.32m x 2.44m.

4 Results

4.1 Implementing forces in Matlab

The three dimensional system to be analyzed is the following

$$m\boldsymbol{a} = \boldsymbol{F}_G + \boldsymbol{F}_D + \boldsymbol{F}_L + \boldsymbol{F}_S \tag{10}$$

where $\boldsymbol{a}$ is the acceleration. In three dimensions the velocity can be described as illustrated in figure 4. Two relevant coordinate systems that could be useful for this system would be either a coordinate system that move along with the ball or a fixed coordinate system - the later was chosen in this report. These coordinate systems are expressed in equations 11 for coordinates that follows the trajectory of the ball and 12 which describe the movement of the ball

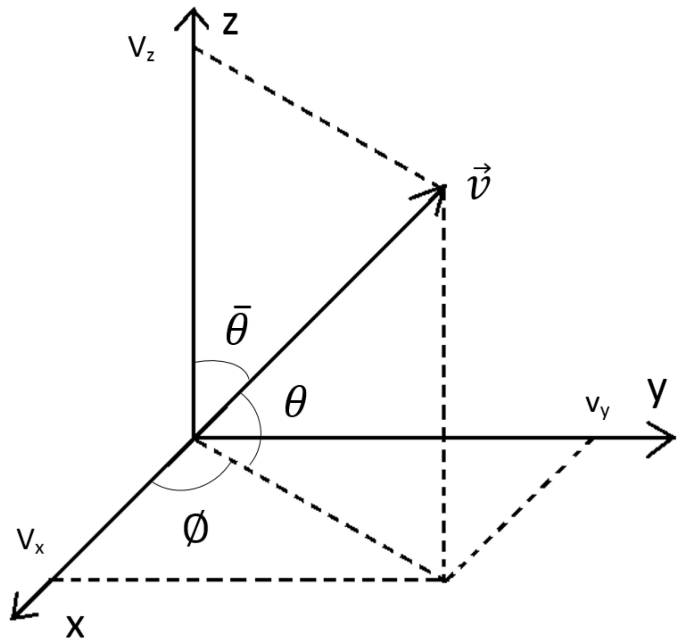

Figure 4: Definition of the angles associated with the velocity vector.

in relation to a fixed coordinate system,

$$\begin{aligned}\boldsymbol{e}_v &= \sin\bar{\theta}\cos\phi\boldsymbol{e}_x + \sin\bar{\theta}\sin\phi\boldsymbol{e}_y + \cos\bar{\theta}\boldsymbol{e}_z \\ \boldsymbol{e}_l &= -\cos\bar{\theta}\cos\phi\boldsymbol{e}_x - \cos\bar{\theta}\sin\phi\boldsymbol{e}_y + \sin\bar{\theta}\boldsymbol{e}_z \\ \boldsymbol{e}_s &= -\sin\phi\boldsymbol{e}_x + \cos\phi\boldsymbol{e}_y\end{aligned} \tag{11}$$

and

$$\begin{aligned}\boldsymbol{e}_v &= v_x\boldsymbol{e}_x + v_y\boldsymbol{e}_y + v_z\boldsymbol{e}_z \\ \boldsymbol{e}_l &= -\frac{v_xv_z}{vv_\perp}\boldsymbol{e}_x - \frac{v_yv_z}{vv_\perp}\boldsymbol{e}_y + \frac{v_\perp}{v}\boldsymbol{e}_z \\ \boldsymbol{e}_s &= -\frac{v_y}{v_\perp}\boldsymbol{e}_x + \frac{v_x}{v_\perp}\boldsymbol{e}_y.\end{aligned} \tag{12}$$

In equation 12 new notations were introduced which are defined as $v = \sqrt{v_x^2 + v_y^2 + v_z^2}$ and $v_\perp = \sqrt{v_x^2 + v_y^2}$. Implementing this coordinate system at equation 10 and solving for the three components of the acceleration yields

$$\begin{aligned}a_x &= -\beta v\Big(C_Dv_x + \frac{C_Lv_xv_z + C_Svv_y}{v_\perp}\Big) \\ a_y &= -\beta v\Big(C_Dv_y + \frac{C_Lv_yv_z - C_Svv_x}{v_\perp}\Big) \\ a_z &= \beta v\big(-C_Dv_z + C_Lv_\perp\big) - g.\end{aligned} \tag{13}$$

The three coefficients (drag, lift and sideways) can be expressed, in accordance with [8], as

$$\begin{aligned}C_D &= -\left(\frac{\big(a_z + g\big)v_z + \big(a_xv_x + a_yv_y\big)}{\beta v^3}\right) \\ C_L &= \frac{\big(a_z + g\big)v_\perp^2 - \big(a_xv_x + a_yv_y\big)}{\beta v^3v_\perp} \\ C_S &= \frac{a_yv_x - a_xv_y}{\beta v^2v_\perp}.\end{aligned} \tag{14}$$

The velocity and acceleration can be described as the derivative and second derivative of the position as

$$a_x = \frac{dv_x}{dt} = \frac{d^2x}{dt^2} \tag{15}$$

and in the same way for y and z. This is a coupled second-order nonlinear differential equation for the position that need to be solved numerically for a chosen generalized coordinate system, which has been chosen to be the coordinate system described in equation 12. These equations can directly be implemented in MatLab in a while-loop. In the while-loop the positions, velocities and the drag-, lift,- and sideways coefficients are given some initial conditions and the first step is to calculate the acceleration with equation 13. When the acceleration has been calculated this will enable the program to calculate new positions, velocities and the drag-, lift,- and sideways coefficients. When this is done, the first loop is done and the program will loop through these equations until the z-position of the ball is equal to zero, i.e. when the ball has returned to the ground again. Comparisons will be made for no spin ($C_L = C_S = 0$) and a ball with spin ($C_L \neq 0, C_S \neq 0$).

4.2 Implementing a football freekick in Matlab

Football is very unpredictable and many components of the game can't be implemented into Matlab without simplifying the situation to much. The approximations that have been made regarding the nature of football is described in the following sections.

4.2.1 The wall

In order to help the goalkeeper, the defending team make a number of players form a wall to protect the goal. Most often the players in the wall are placed in order to protect the post closest to the ball. The number of players in a wall is varied depending on where the free kick is taken and on the strategy from the two teams. The wall is, according to FIFA regulations, placed 7 meter from the ball. In many real life free kicks the forward tries to curl the ball around the wall, however in order to exclude problem regarding the width of the wall the wall was assumed to be infinitely long. The wall was placed to be perpendicular to the line going from origin to the left post, intersecting at a distance 7 meters from the origin.

4.2.2 The goalkeeper

In reality all balls that are within the boundaries of the goal will not score due to the fact that a goalkeeper try to save the free kicks. It will be impossible to make an algorithm that show close resemblance with a real life goalkeeper, however by noting some intuitive facts of goalkeepers it's possible to come up with some properties that can be included in the algorithm. First of all, it's more difficult for the goalkeeper to catch a ball close to the boundaries of the goal and it's also more difficult to catch a fast ball than a slow. The goalkeeper will also sometimes make mistakes and therefore balls that have low velocity and/or bad placement will sometimes reach the back of the net. These three properties have been implemented into the program by assuming that the probability of scoring can be described as a linear combination of the ball position and the mistakes by the goalkeeper as

$$P = P_{pos} + P_m, \qquad P_{pos} = \alpha P_x P_z. \tag{16}$$

P_x and P_z are the probabilities to score based on the position of the ball in the goal. These probabilities are defined as half circles with the radius $r_x = W/2$ and $r_z = H/2$. α is a normalization constant. The value of α was determined by trial and error to be 900 in order to make a reasonable number of sufficiently placed balls reach the goal. This approximation will make it more plausible to score closer to the boundaries of the goal while having the goalkeeper save almost all shots in the center of the goal. P_m is the probability that the goalkeeper make a mistake and this factor is modeled to vary between 0 and 0.25 for each shot. If the probability of scoring, P, reached above a critical P, P_c then the ball fulfilled the criteria for scoring. The last property included in the scoring probability was the speed of the ball. It was found to be much better to decide for a critical time for each shot instead of a critical speed. If it took the ball longer than a critical time to reach the goal, then the goalkeeper would have enough time to save the ball. The critical time was determined by trial and error to be $t_c = 2.1$ s. This time is higher than expected and discussed in the discussion.

4.3 Simulation results

The results are divided into two subsections. In the first subsection the difference between different approximations will be illustrated and in the second section Monte Carlo simulations will be performed in order to validate this trajectory model in comparison with the real data. More details about the Monte Carlo method will be given in section 4.4.

4.3.1 Different models

Four approximations are compared with each other in figure 5 with input from table 2. The left post of the goal was placed at (22.5, 25.85) keeping the initial ball position to be in the origin. The blue line illustrate the trajectory when the ball is subjected to all forces mentioned in this report and should therefore be viewed as a reference in comparison with the others. The coefficients of the blue line were used for the Monte Carlo simulations. In the other models one or several forces have been neglected and it's evident that there is a dramatic difference between these models.

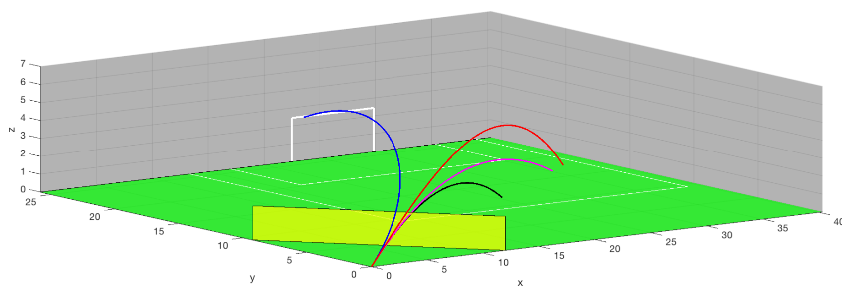

Figure 5: Ball paths for four different approximations. The coefficients used is tabulated in table 2. Initial velocities for all balls: $v_x = 27.3$ m/s, $v_y = 7.7$ m/s and $v_z = 7.1$ m/s.

Input	Blue line	Red line	Black line	Purple line
C_S	0.4	0	0	0
C_L	0.2	0.2	0	0
C_D	0.3	0.3	0.3	0

Table 2: Input parameters used in Matlab to create figure 5

4.4 Monte Carlo simulations

If the striker would execute all free kicks with perfect precision the results would look like figure 6. For these six different initial conditions all 50,000 free kicks ended up in one of the two corners. In reality it would be impossible for the forward to perform all free kicks with equal initial velocities and therefore it's necessary to include an uncertainty

$$\boldsymbol{v} = \big(v_x + \delta\big)\boldsymbol{e}_x + \big(v_y + \delta\big)\boldsymbol{e}_y + \big(v_z + \delta\big)\boldsymbol{e}_z. \tag{17}$$

The uncertainty has been randomized in the interval within $\pm\delta$. The uncertainty was varied between 0 m/s, corresponding to perfect conditions in figure 6, to 2.5 m/s. The trajectories of $\delta = 0.5$, 1 and 2 are illustrated in figures 7, 8 and 9. For all these figures the blue lines are the trajectories of the balls that score, the purple lines are the trajectories of the balls being saved, the black lines miss the goal and the red lines are the balls that hit the wall. The effect of changing δ is illustrated in figure 10 from 21 simulations of 50,000 free kicks each. According to [7] the conversion rate from the location investigated in this report would be around 4%. The experimentally obtained conversion rate intersect with the average data when $\delta = 2$. Three different initial conditions for each corner were implemented in order to get a broader variety of trajectories. These initial conditions are illustrated in table 3.

Placement	v_x	v_y	v_z	v
Top left	21.5	10.4	8.4	25.3
Top left	22.8	10.8	7.5	26.3
Top left	23.1	11.4	7.1	26.1
Top right	27.3	7.7	7.1	29.2
Top right	27.0	8.1	7.1	29.1
Top right	26.8	7.9	7.4	28.9

Table 3: Initial velocities (m/s) used in the Monte Carlo simulations.

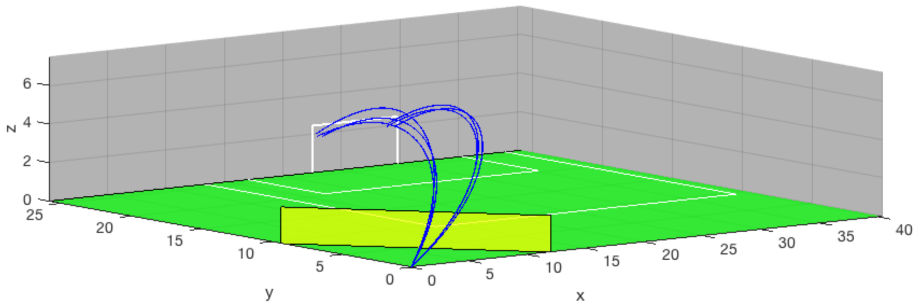

Figure 6: Trajectories of the balls with no uncertainties, $\delta = 0$. $C_S = 0.4$, $C_L = 0.2$, $C_D = 0.3$.

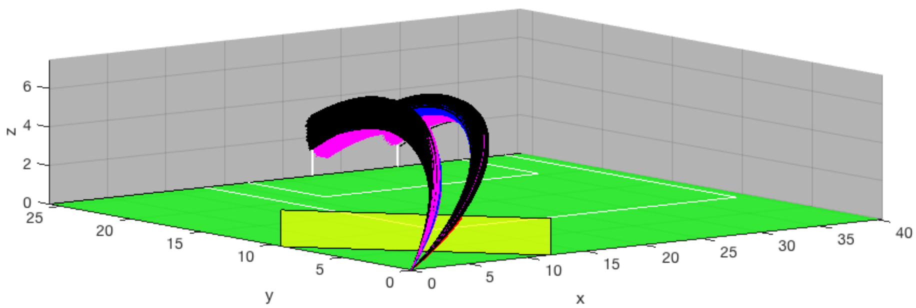

Figure 7: Trajectories of the balls that with $\delta = 0.5$. $C_S = 0.4$, $C_L = 0.2$, $C_D = 0.3$.

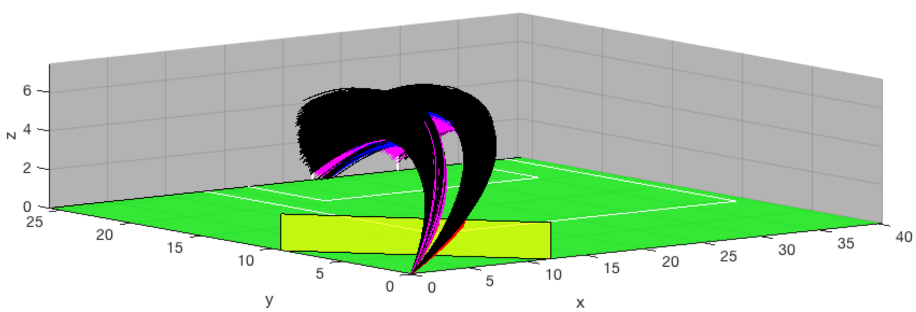

Figure 8: Trajectories of the balls with $\delta = 1.0$. $C_S = 0.4$, $C_L = 0.2$, $C_D = 0.3$.

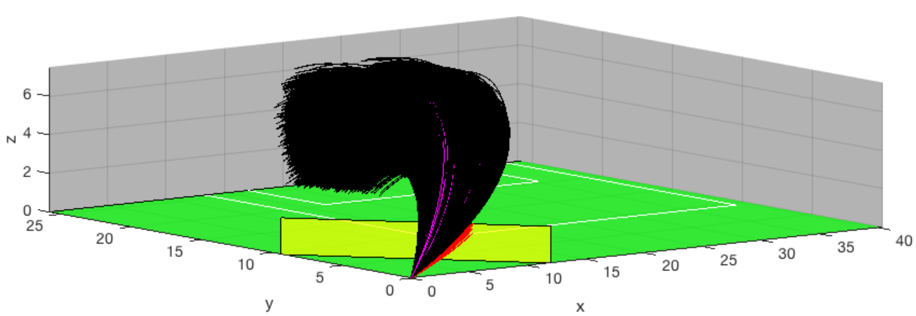

Figure 9: Trajectories of the balls with $\delta = 2.0$. $C_S = 0.4$, $C_L = 0.2$, $C_D = 0.3$.

From the ball trajectories, although not noticeable in the figures in this report, it was clear that the goal distribution changed significantly. At $\delta = 0$ all balls that scored where placed in the two top corners, however this changed as the precision decreased. In figure 11 the distribution of all balls scored for different values of δ is illustrated.

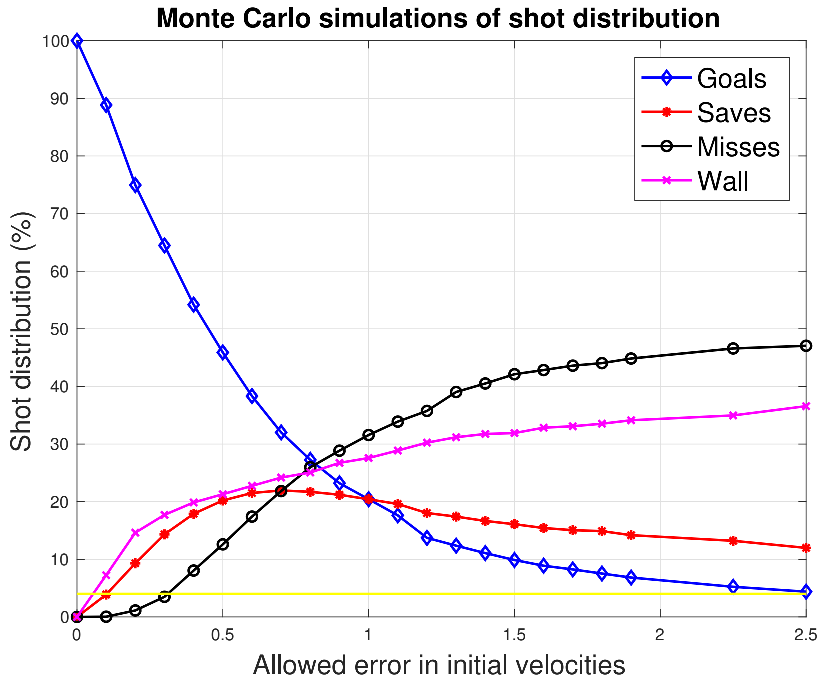

Figure 10: Illustration on the effects of changing δ. The yellow line is the average conversion rate for this location from all top European leagues at this location. [7]

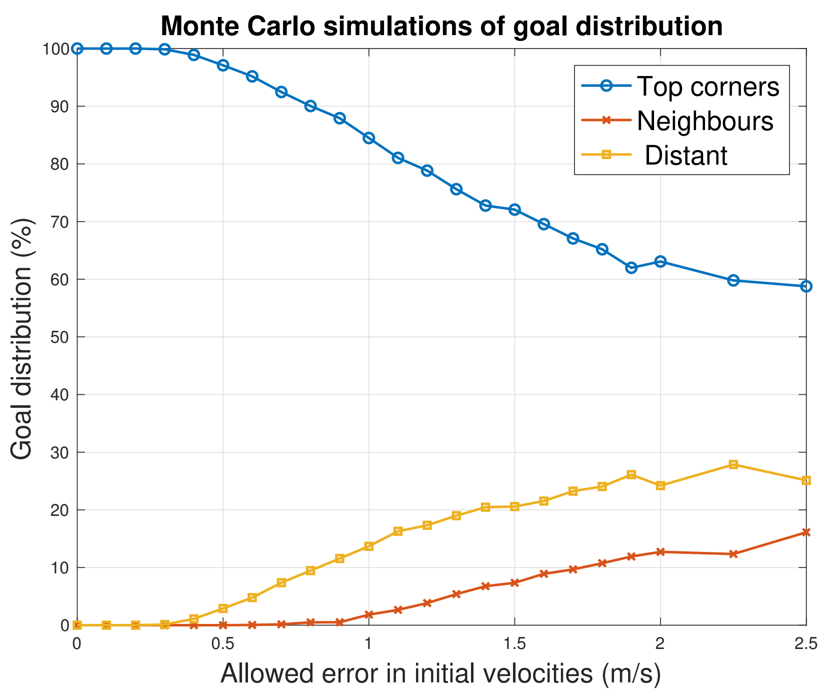

Figure 11: The change of δ shows that the goals are scored in different locations of the goal.

5 Discussion

By investigating figure 6 it was evident that the different models give very different answers. It was clear that the Magnus effect and the air resistance was observed and that it had a significant impact on the results as expected. It's also important to keep in mind that the coefficients was chosen arbitrarily (not C_D since this value was based on several reports mentioned earlier). It's very probable that the values of C_L and C_S was not reasonable since the curl and lift differed more than expected by intuition. In further studies the effects of these coefficients should be investigated further.

The experimental conversion rate matched the average conversion rate when $\delta = 2.5$ m/s, however it's not possible to say that football players have such bad control of their free kicks. It should be noted that in reality the forward usually try to bend the ball around the wall in order to have a more powerful shoot without missing the goal. This was not allowed in this model since there was some computational error that occurred that couldn't be addressed without spending to much time on this report. The goalkeeper approximation seem however to be a somewhat good approximation. I would however not expect that so many balls would score at the distant regions as they did. All balls scored in the distant regions where scored at two regions located at the bottom left. The balls to the right travelled further and they reached the goal in more time than those to the left and therefore these regions wasn't that successful. All times where however longer than expected and therefore the cut off time needed to be increased from 1 s which I thought was reasonable to more than double. This problem highlight that although the trajectories seem to be correct, all balls travel for longer times than expected and therefore this model have a large fault. The speed of the balls could be increased by curling around the wall, however this change would probably not be enough.

In reality there are many uncertainties regarding the condition of the forward, the goalkeeper, the grass, air and so on. The location of the wall should be at least 7 m from the ball, however the referee will only estimate this distance by walking steps that are approximately 1 meter each. It's much more difficult to score a goal when the ball is closer to the ball since this demand higher v_x and less v_x or v_y. This would imply that the trajectory of the ball would make the ball easier to catch for the goalkeeper since the ball will be in the air for a longer period of time. For both players there are both physiological effects such as lactic acids and fatigue and also psychological aspects that can't be investigated with this model. The probability of having the goalkeeper making a mistake was incorporated into the model, however this is a very rough and exaggerated approximation. Most goalkeepers make mistakes rarely, however in order to observe this effect, the error probability was exaggerated. This caused more balls to be scored at distant regions than what was expected. More sophisticated models for both the goalkeeper, the wall and the forward can and should be implemented if this model would be developed further in the future.

In the Monte Carlo simulations $C_S = 0.4$ was used for all free kicks. In reality it would be impossible to have identical curl on the ball if other initial conditions where changed. It would for example be more probable to involve less curl if the forward shots over the wall than if the forward bend the ball around the wall. A sideways coefficient of 0.4 give a slightly more dramatic curl than expected. This value was used since it show a great difference in comparison with the approximations with no curl. A more reasonable sideways force would probably be obtained closer to $C_S = 0.2$. Since the balls had more curl than expected the trajectories where also longer than expected. This problem might be the biggest contribution for the too long flight time problem. In further studies this should be investigated.

The purpose of this report was to investigate why it's so difficult to score from a direct free kick. This is clearly highlighted by figures 10 and 11. It's evident that the free kick system has a high sensibility for the initial conditions. Already at $\delta = 0.5$ m/s more than half the free kicks failed. One interesting observation is that almost 15% of all goals scored at $\delta = 1$ m/s occurs at a position that is not at all the position that the forward aimed at. For beginners this might be an interesting feature of their lack of precision, however in the highly professionalized top leagues in Europe I found it very unlikely that a forward aiming at a specific top corner would miss this position with two meters and still have a sufficient speed to score. Psychology is however a key feature of sports and the Swedish footballer Kim Källström claimed that during the penalty shootout of the FA cup 2014, he was aiming at the top left corner whereas the goal was scored in the bottom right corner. He claimed that he wasn't nervous but his body didn't respond like he was used to due to the extreme pressure from the audience, his club and himself. [13] This example could give some insight that professional footballers might be more controlled by their emotions than one might expect and this observation make several aspects of sports almost impossible to analyze with mathematics as in this report. In order to make a perfect prediction the mathematics should involve the specific time of the free kick since players get tired throughout the match, the specific weather conditions, the importance of this specific game, the specific goalkeeper and forward and their individual mental status and how much pressure there are on them. Luck is also a influence of the outcome of a free kick. These aspects make free kicks very difficult to predict and therefore future studies are needed.

It would have been interesting to investigate the Magnus effect in the Lagrangian- or Hamiltonian formalism. This is done in [11], however I don't have access to this article and I couldn't find any other good references for this topic. For future research (maybe future projects in this course) it would be interesting to either look into this article or find another one which give insight of the other formalism's. In this report the ball was given an initial velocity whereas in reality the footballer would need to make impact with its foot on the football in order to transfer energy and cause the ball to move. I have restricted this project to only treat the movement of the ball and therefore this was not included either, however for future studies a Lagrangian impact model is presented in [9] which could be useful when expanding this project further. If this is done in the future it would be very interesting to find a more realistic error in the initial velocities. More realistic error in the initial velocities could also perhaps be found when searching information in articles regarding training and physiology.

5.1 Conclusion

It was concluded that the margin of error is very small for the initial velocities of free kicks. If the forward have a shot uncertainty of only 0.5 m/s in all directions then more than half of the shots will fail in the model used in this report. It was also concluded that the Magnus effect have great influence on the behaviour of a football. It was finally also concluded that the model used in this report for the goalkeeper is too simple for proper analysis since there are properties of the game that is difficult to analyze such as psychology.

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